Analysis of full-QCD and quenched-QCD lattice propagators

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Abstract. Recent lattice-QCD results for the dressed-gluon propagator are used within the quark Dyson-Schwinger equation to determine the gluon-quark vertex dressing necessary to reproduce the lattice-QCD results for the dressed-quark propagator. Both quenched and full QCD lattice simulations, for a range of low quark current masses, are analyzed. The chiral extrapolation is made through this continuum DSE form. Resulting chiral and physical pion observables are investigated.

Keywords: Dyson-Schwinger equation, chiral symmetry, dressed quark-gluon vertex, lattice-QCD.
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INTRODUCTION

Lattice-QCD simulations address hadronic observables via matrix elements of currents and sources; necessary approximations and truncations introduce systematic errors from: a finite volume of discretized space-time, unphysically large current quark masses, and the quenched approximation. While steady progress is being made in the reduction of such errors, it is useful to gain insight into dominant field theory mechanisms by comparing to hadronic results from covariant modeling of continuum QCD. The Dyson-Schwinger equations (DSEs), the equations of motion of the theory, provide such an opportunity. Here systematic errors can arise from truncations that replace high-order correlations by infrared phenomenology of low-order n-point functions. Symmetries can provide significant quality control. The dressed quark and gluon propagators and low-order vertices of the theory are important elements of the kernels needed for the bound state equations: Bethe-Salpeter equation (BSE) for mesons and Faddeev equation for baryons. Prior to the last few years, one had only the known ultra-violet behavior of the n-point functions as a guide.

Recently, full-QCD lattice results for gluon [1] and quark [2] propagators have been made available; these are two of the three nonperturbative quantities linked by the quark propagator DSE. Phenomenological information on the third quantity, the dressed quark-gluon vertex $\Gamma^a_\nu(k,p)$, can therefore be inferred. We have previously carried out such an analysis of quenched-QCD lattice propagators [3] and the quark-gluon vertex [4]; the procedures and technical details employed here are described in Ref. [3]. Here, and in the lattice simulations, Landau gauge and the Euclidean metric is used. Briefly, the approach is the following. From the renormalized quark DSE, we have

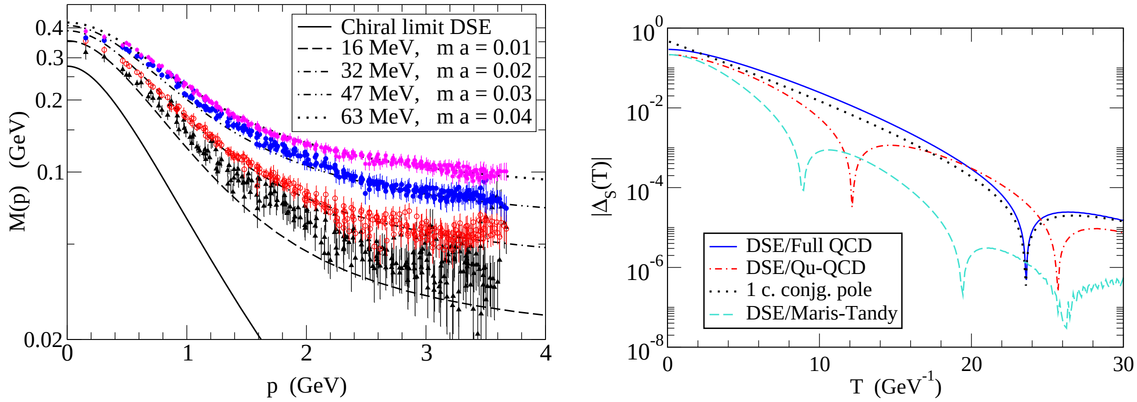

FIGURE 1. *Left Panel*: Full-QCD lattice results [2] reproduced from the full-QCD lattice gluon propagator [1] via the continuum DSE. Note vertical log scale. Values of lattice m and ma are shown; *Right Panel*: Fourier transform of the Dirac scalar part of the chiral $S(p)$. Cusps indicate confinement.

$S^{-1}(p) = Z_2\, i\gamma\cdot p + Z_4\, m(\mu) + \Sigma'(p,\Lambda)$ and the regulated self-energy is

$$\Sigma'(p,\Lambda) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\, \frac{\lambda^a}{2}\gamma_\mu\, S(q)\, \Gamma^a_\nu(p-q,p) \quad . \tag{1}$$

Here Λ is the regularization mass scale, the $Z_i(\mu^2,\Lambda^2)$ are the usual constants from renormalization at scale μ. Two are required so that as $p^2 \to \mu^2$, we have $S^{-1}(p) \to i\gamma\cdot p + m(\mu)$. The lattice results for $S(p)$ are given in terms of $S(p) = Z(p^2,\mu^2)/\,(i\gamma\cdot p + M(p^2))$. We compare by solving (1) with the kernel factors $Z_1\, g^2\, D_{\mu\nu}(k)\, \Gamma^a_\nu(k,p)$ replaced by $D^{\rm lat}_{\mu\nu}(k)\, \frac{\lambda^a}{2}\, \gamma_\nu\, \Gamma_1(k^2)$. Here $D^{\rm lat}_{\mu\nu}(k)$ is a fit to the lattice gluon propagator and the quantity $\Gamma_1(k^2)$ is a phenomenological vertex amplitude determined so that the DSE solution for $S(p)$ fits the lattice data. In general the transverse vertex has eight amplitudes and a dependence upon two momenta; however the data being fitted here do not warrant more. We note that one may identify the kernel as $4\pi\alpha_{\rm eff}(k^2)\, D^0_{\mu\nu}(k)$ where $D^0_{\mu\nu}$ is the 0$^{\rm th}$ order gluon propagator. We ensure that the leading log behavior of all quantities conform to the 1-loop renormalization group behavior of QCD.

RESULTS

TABLE 1. Chiral condensate and pion decay constant (chiral f^0_π, physical f_π) from the lattice-guided DSE kernel.

	Expt	Full-QCD	Qu-QCD
$\langle \bar{q}q\rangle_{\mu=1\ \rm GeV}$	-(0.24 GeV)^3	-(0.23 GeV)^3	-(0.19 GeV)^3
f^0_π	0.090 GeV	0.072 GeV	0.063 GeV
f_π	0.092 GeV	0.075 GeV	0.066 GeV

Our fit to the full-QCD $D^{\rm lat}_{\mu\nu}(k)$ uses the "model A" form previously employed for quenched data [5] but now with $N_f = 3$ and the new parameter values: $A = 3.25$, $\Lambda_{\rm g} = 0.54$, $\alpha = 1.15$ and $Z_{\rm g} = 1.22$. Our fit gave priority to $M(p)$ for the four available values of lattice ma shown, along with the results, in the left panel of Fig. 1. The parameterized form used for $\Gamma_1(k^2)$ is the same as we have previously used [3] for the quenched lattice case. The (dimensionless) parameters of $\Gamma_1(k^2)$ found here for the full-QCD case are: $a_1 = 4.5, a_2 = 2.1, a_3 = 18.1, b = 0.31$. The result of the chiral extrapolation provided by the DSE kernel is also shown. The lattice current masses are equally spaced and, in the region $p \gtrsim 3GeV$, the lattice results are only approximately so; the $ma = 0.01$ case deviates most from the pattern. The DSE fit has been made with the constraint that the $M(p,m)$ approach the correct ratio.

If a propagator of a field theory in Euclidean metric violates the Osterwalder-Schrader axiom of relection positivity [6], then this is a sufficient condition for confinement of the corresponding excitation [7]. In the right panel of Fig. 1 we display the magnitude of the Fourier transform of $\sigma_s(p_4, \vec{p} = 0)$, the Dirac scalar amplitude of the chiral limit quark propagator. For a free particle with mass m, $\Delta_S(T) \propto \exp(-mT)$. The cusps indicate changes of sign and thus confinement in both quenched and full-QCD. The dotted line corresponds to a propagator that has a single pair of complex conjugate poles at $p^2 = -0.309^2 \pm i0.192^2$ GeV^2. The dashed curve corresponds to the ladder-rainbow model that describes a large variety of light quark observables [8].

The form of the deduced DSE kernel allows a chiral symmetry-preserving Bethe-Salpeter kernel to be obtained as in the ladder-rainbow case [9]. This produces the chiral physics observables shown in Table 1. Full-QCD evidently produces an acceptable condensate, while the quenched approximation underestimates by a factor of two [3]. The values of f_π, both chiral and physical, are marginally improved by full-QCD but they remain about 15% too low.

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